International Conference

## Multiscale Modeling in Fluid Mechanics and Fluid-Structure Interaction

October 7-11, 2019, Vilnius, Lithuania

## Abstracts

Vilnius, 2019

### TABLE OF CONTENTS

1 Transcatheter and Minimally-Invasive Heart Valve Interventions: Current Approaches and Future Perspectives

Audrius Aidietis, Diana Zakarkaite, Sigita Aidietiene, Agne Drasutiene, Kestutis Rucinskas

- 2 Microstructure in single crystals and polycrystals John Ball
- 3 Robust multiscale modeling of blood flows *Cristóbal Bertoglio*
- 4 On weakly singular kernels arising in equations set on a graph, modelling a flow in a network of thin tubes

Éric Canon, Frédéric Chardard, Grigory Panasenko, Olga Štikonienė

- 5 Asymptotic analysis and hybrid dimension models for the flows in thin tube structures Éric Canon, Frédéric Chardard, Grigory Panasenko, Konstantinas Pileckas, Olga Štikonienė
- 6 Global existence of weak solutions for a fluid beam interaction system *Jean-Jérôme Casanova*, *Céline Grandmont*, *Matthieu Hillairet*
- 7 Numerical solution of the viscous flows in a network of thin tubes: equations on the graph *Frédéric Chardard*, Éric Canon, Grigory Panasenko, Olga Štikonienė
- 8 Personalized Computational Modeling of Transapical Mitral Valve Repair <u>Gediminas Gaidulis</u>, Audrius Aidietis, Rimantas Kačianauskas
- 9 On the Motion of a Rigid Body with a Liquid-Filled Interior Cavity <u>Giovanni P. Galdi</u>
- 10 Mixed-Dimensional Coupling for Time-Dependent Wave Problems Dan Givoli, Hanan Amar, Daniel Rabinovich
- 11 Asymptotic analysis of the viscous flow through a pipe and the derivation of the Darcy-Weisbach law

Eduard Marušić-Paloka

- 12 The FSI framework for the numerical modeling of the arterial blood flow and its application in the clinical case scenario *Edgaras Misiulis, Algis Džiugys, Robertas Navakas, Gediminas Skarbalius*
- 13 Existence and regularity for weak solutions for a fluid interacting with a non-linear shell in 3D *Boris Muha, Sebastian Schwarzacher*
- 14 Biomechanics of arterial thrombosis Dmitry Nechipurenko, Valeria Kaneva, Mikhail Panteleev, Fazly Ataullakhanov
- 15 Effects of rough boundary and boundary conditions on the lubrication process with micropolar fluid

Igor Pažanin

- 16 Mathematical models for the heart and the circulation *Alfio Quarteroni*
- 17 Rigorous justification of the asymptotic model describing a nonsteady micropolar fluid flow through a thin curved pipe *Marko Radulović, Igor Pažanin*
- 18 Cardiovascular modeling and simulations. Applications to some clinical studies *Adélia Sequeira*
- 19 Spatial asymptotics of time dependent Stokes problems *Maria Specovius, Konstantinas Pileckas*

- 20 Control of fluid-structure interaction problems with medical applications <u>Ruxandra Stavre</u>
- 21 Incompressible viscous fluid interacting with a nonlinear thermoelastic plate *Srđan Trifunović, Ya-Guang Wang*
- 22 Method of asymptotic partial decomposition with discontinuous junctions. Finite volume implementation <u>Marie-Claude Viallon</u>, Grigory Panasenko
- 23 On the Equations of Nonlinear Single-Phase Poroelasticity *C. J. van Duijn*, <u>Andro Mikelić</u>
- 24 A Multi-scale Model for Magnetorheological Suspensions Bogdan Vernescu, Grigor Nika
- 25 Fluid flow in a flexible pipe with layered elastic walls <u>German Zavorokhin</u>, Sergei Nazarov, Vladimir Kozlov

#### **Participants**

26

#### **Index of Authors**

27

#### Transcatheter and Minimally-Invasive Heart Valve Interventions: Current Approaches and Future Perspectives

Audrius Aidietis, Diana Zakarkaite, Sigita Aidietiene, Agne Drasutiene, Kestutis Rucinskas Vilnius University, Vilnius University Hospital Santaros Klinikos, Lithuania Audrius.Aidietis@santa.lt

#### **Objectives:**

1. To provide an overview and a clinical perspective on novel transcatheter heart valve therapies as a single source of introductory information and fundamental understanding of the basic clinical science, pre- and post-procedure assessment as well as management of these modern therapies. These technologies have transformed the management of valvular heart disease for patients who are unfavourable candidates for surgery, especially for elderly patients at high surgical risk.

Success of transcatheter aortic valve implantation (TAVI) devices over the last decade has generated a considerable interest in new transcatheter technologies specifically designed for repair and replacement of the native mitral, tricuspid and pulmonary valve.

2. To present our results and clinical experience of new technologies which we used in our hospital. **Methods and Results:** 183 patients have been treated with TAVI in our institution between January 2009 and January 2019. Mortality, for those at high risk, was only 5.2% & 11.4% at 30 d & 1 y accordingly. The main cause of cardiac death was heart failure (54%). Independent clinical predictors of death were increased by age, atrial fibrillation, pulmonary disease & LVEF <50%.

Between November 2009 and June 2019 we treated 107 symptomatic patients with severe mitral regurgitation due to leaflet flail/prolapse. We used a newly developed NeoChord DS1000 system: a device designed for transapical implantation of artificial chordae tendineae (neochords) to correct mitral regurgitation on the beating heart without cardiopulmonary bypass with minimally invasive technique. 50 initial patients were enrolled in the NeoChord TACT and the NeoChord REGISTRY Trials. According to primary endpoint, patients were stratified into 2 groups: group A – 85 (79,4%) patients with successful durable MV repair; group B – 22 (20,6 %) patients with MV repair failure (recurrence of severe MR or reintervention due to this matter). Group B patients had significantly larger LV diameters, LA volumes, greater MV tenting volume, higher systolic pulmonary pressure values and they had achieved lower coaptation length of mitral valve leaflets compared with group A.

On the 5th October in 2018, the first-in-human transcatheter treatment of secondary tricuspid regurgitation (TR) with a novel Minimally Invasive Annuloplasty (MIA) System by transjugular approach was performed in Vilnius University Hospital. The first and the following 7 patients have been enrolled as a part of STTAR Trial. Anchors connected with suture were implanted percutaneously into the posterior tricuspid valve annulus. Upon the device deployment and anchor approximation an average reduction of 40% in tricuspid valve area and 1-2+ grade reduction in TR along with functional improvement was observed in all 8 patients. There were no serious adverse events related to the device implantation.

#### **Conclusions:**

1. This comprehensive evaluation of all the patients treated with this innovative technology provides reassurance regarding the long-term clinical efficacy of TAVI as well as gives insight into the evolution of our programme with time.

2. The NeoChord procedure is a safe and effective therapeutic intervention for severe MR prior to onset of annular dilation and LV remodelling. Furthermore, this procedure provides good long-term results in selected patients.

3. The safety and feasibility of tricuspid valve repair with this novel annuloplasty system (MIA system) have been demonstrated. Pacemaker leads are not a contraindication for device implantation with this system. Further studies needed to confirm these data in larger cohorts of patients.

International Conference Multiscale Modeling in Fluid Mechanics and Fluid-Structure Interaction Vilnius, Lithuania, October 7–11, 2019

## Microstructure in single crystals and polycrystals

John Ball Heriot-Watt University, Edinburgh, Scotland, jb101@hw.ac.uk

The talk will discuss the description of microstructure in single crystals and polycrystals due to martensitic phase transformations, in which the underlying crystal lattice changes shape at some critical temperature, or applied loads. This involves understanding compatibility both in individual crystal grains and across grain boundaries, for which generalizations of the Hadamard jump conditions may be useful (as in joint work with C. Carstensen). International Conference *Multiscale Modeling in Fluid Mechanics and Fluid-Structure Interaction* Vilnius, Lithuania, October 7–11, 2019

## **Robust multiscale modeling of blood flows**

Cristóbal Bertoglio Bernoulli Institute, University of Groningen, Groningen, Netherlands, c.a.bertoglio@rug.nl

In blood flows, it is unfeasible to model, measure and perform numerical simulations of the complete or large portions of the cardiovascular system. Therefore, such computations are focused on specific regions of interests depending on the posed clinical question. Still, the effect of the rest of the cardiovascular system needs to be reliably taken into account in terms of boundary conditions, leading to coupled problems at different spatial scales. After an introduction to the topic, in this talk I will review my work which as tackle several associated challenges: backflow stabilization on open boundaries [1, 2, 3], numerical analysis of coupled 0D-3D fluid(-solid) problems [4], and asymptotic model reduction [5]. I will also describe the how to compensate for inaccuracies in the computational domain itself by more advanced boundary conditions [6].

- [1] C. Bertoglio and A. Caiazzo. A tangential regularization method for backflow stabilization in hemodynamics, *J.Comp.Phys.*, **261**:162–171, 2014.
- [2] C. Bertoglio and A. Caiazzo. A Stokes-residual backflow stabilization applied to physiological flows., *J.Comp.Phys.*, 313:260–278, 2016.
- [3] C. Bertoglio, A. Caiazzo, Bazilevs, M. Braack, V. Gravemaier, A. Marsden, M. Moghadam, O. Pironneau, I. Vignon, and W. Wall. Benchmark problems for numerical treatment of backflow at open boundaries., *Int.J.Num.Meth.Biomed.Eng.*, 34:e2918, 2018.
- [4] C. Bertoglio, A. Caiazzo, and M. Fernández. Fractional-step schemes for the coupling of distributed and lumped models in hemodynamics., SIAM J.Sci.Comp., 35:B551–B575, 2013.
- [5] C. Bertoglio, C. Conca, D. Nolte, G. Panasenko, and K. Pileckas. Junction of models of different dimension for flows in tube structures by Womersley-type interface conditions.
- [6] D. Nolte and C. Bertoglio. Reducing the impact of geometrical uncertainties in blood flows simulations using velocity measurements., *Int.J.Num.Meth.Biomed.Eng.*, e3203, 2019.

## On weakly singular kernels arising in equations set on a graph, modelling a flow in a network of thin tubes

 Éric Canon<sup>1</sup>, Frédéric Chardard<sup>1</sup>, Grigory Panasenko<sup>1,2</sup>, Olga Štikonienė<sup>2</sup>
 <sup>1</sup> Univ Lyon, UJM-Saint-Etienne, CNRS, Institute Camille Jordan UMR 5208, SFR MODMAD FED 4169, F-42023, Saint-Etienne, France
 <sup>2</sup> Institute of Applied Mathematics, Vilnius University, Vilnius, Lithuania eric.canon@univ-st-etienne.fr, frederic.chardard@univ-st-etienne.fr, grigori.panasenko@univ-st-etienne.fr, olga.stikoniene@mif.vu.lt

This talk follows the talk by Frédéric Chardard entitled: Numerical solution of the viscous flows in a network of thin tubes: equations on the graph. These equations are set on a 1D graph and were obtained by letting the diameters of the tubes tend to zero in some asymptotic process. They are characterized by a convolution in time in the diffusion operator, with a weakly singular kernel in time that are computed from the solution of local heat equations (with Dirichlet Conditions) in 2D domains that represent the cross sections of the tubes of the initial network. This talk is more about these kernels: theoretical results and numerical computations of the kernels. We obtain in particular asymptotic expansions for small times in different way: for smooth cross section inspired by techniques developed by Gie, Hamouda, Jung and Temam (Singular Perturbations and Boundary Layers, Springer), or for specific cross sections (rectangles, disks, equilateral triangles) with specific techniques. Possibly, we will also mention some work in progress for kernels with different boundary conditions, or use of the same kernels in other contexts.

## Asymptotic analysis and hybrid dimension models for the flows in thin tube structures

Éric Canon<sup>1</sup>, Frédéric Chardard<sup>1</sup>, Grigory Panasenko<sup>1,2</sup>, Konstantinas Pileckas<sup>2</sup>, Olga Štikonienė<sup>2</sup> <sup>1</sup> Univ Lyon, UJM-Saint-Etienne, CNRS, Institute Camille Jordan UMR 5208, SFR MODMAD FED 4169, F-42023, Saint-Etienne, France

<sup>2</sup> Institute of Applied Mathematics, Vilnius University, Vilnius, Lithuania eric.canon@univ-st-etienne.fr, frederic.chardard@univ-st-etienne.fr, grigori.panasenko@univ-st-etienne.fr, konstantinas.pileckas@mif.vu.lt, olga.stikoniene@mif.vu.lt

Thin structures are some finite unions of thin rectangles (in 2D settings) or cylinders (in 3D settings) depending on small parameter  $\varepsilon \ll 1$  that is, the ratio of the thickness of the rectangle (cylinder) to its length. Viscous flows in such structures are modeled by steady or non-steady Stokes or Navier-Stokes equations stated in thin structures with the no-slip boundary condition at the lateral boundary of the cylinders and with the inflow and outflow conditions with the given velocity on some part of the boundary.

For thin structures an asymptotic expansion of the solution is constructed and justified. It has a form of a Poiseuille (or Womersley) flow within thin cylinders at some distance from the bases while the boundary layers near the ends of the cylinders decay exponentially. The algorithm of construction of the expansion deals with a special Reynolds type problem on the graph for the pressure. This structure of the expansion allows to reduce the dimension within the cylinders at the distance of order  $\varepsilon |\log \varepsilon|$  from the bases of the cylinders and derive the junction conditions between models of different dimensions.

- [1] Panasenko G. P. Asymptotic expansion of the solution of Navier-Stokes equation in a tube structure, *JC.R.Acad.Sci.Paris*, **326**, Série IIb: 867–872, 1998.
- [2] Panasenko G. P. Partial asymptotic decomposition of domain: Navier-Stokes equation in tube structure, JC.R.Acad.Sci.Paris, 326, Série IIb: 893–898, 1998.
- [3] Panasenko G. P. *Multi-Scale Modelling for Structures and Composites*, Springer, Dordrecht, 398 pp., 2005.
- [4] Panasenko G., Pileckas K. Asymptotic analysis of the nonsteady viscous flow with a given flow rate in a thin pipe, *Applicable Analysis*, **91**(3): 559–574, 2012.
- [5] Panasenko G., Pileckas K., Divergence equation in thin-tube structure, *Applicable Analysis*, 94(7): 1450-1459, 2015. doi 10.1080/00036811.2014.933476
- [6] Panasenko G. P., Pileckas K., Flows in a tube structure: equation on the graph, *Journal of Mathematical Physics*, 55: 081505, 2014, doi: 10.1063/1.4891249.
- [7] Panasenko G., Pileckas K. Asymptotic analysis of the non-steady Navier-Stokes equations in a tube structure. I. The case without boundary layer-in-time, *Nonlinear Analysis, Series A, Theory, Methods* and Applications, 122:125-168, 2015. http://dx.doi.org/10.1016/j.na.2015.03.008
- [8] Panasenko G., Pileckas K. Asymptotic analysis of the non-steady Navier-Stokes equations in a tube structure. II. General case, *Nonlinear Analysis, Series A, Theory, Methods and Applications*, 125:582-607, 2015. http://dx.doi.org/10.1016/j.na.2015.05.018

### Global existence of weak solutions for a fluid beam interaction system

Jean-Jérôme Casanova<sup>1</sup>, <u>Céline Grandmont</u><sup>1</sup>, Matthieu Hillairet<sup>2</sup> <sup>1</sup> Inria & Sorbonne University, Paris, France, <sup>2</sup> Montpellier Univesity, Montpellier, France, jean-jerome.casanova@inria.fr, celine.grandmont@inria.fr, matthieu.hillairet@umontpellier.fr

We consider a coupled system of pdes modelling the interaction between a 2D incompressible viscous fluid and an elastic beam located on the upper part of the fluid domain boundary. Our aim is to prove existence of global weak solutions "beyong" possible contact. To do so, we take advantage of the existence of global strong solution for a viscous structure proved in [1] and let the additional viscosity tend to zero. Our scheme is inspired by the one developed in [2] where the global existence of a weak solution is derived for a 2D fluid-solid problem. However, in [2] the solids are viewed as inclusions whose viscosities is infinite. The fluid-solid problem is then approximated by a fluid problem with a different viscosity in the inclusions and in the fluid. The viscosity of the inclusions is then sent to infinity. In contrast, in our case the parabolichyperbolic fluid-structure system is approximated by a parabolic-parabolic one by adding viscosity to the structure. We prove that, up to the extraction of a subsequence, the sequence of solutions of the damped system converges towards a weak solution (in a sense to be defined) of the undamped system. The main difficulties are to define functional and variational frameworks compatible with a possible contact and to prove the  $L^2$  compactness of the fluid and structure velocities again in the case of a possible contact. The strategy is to first obtain compactness of a projection of the fluid and structure velocities and then prove that the space on which the projection is done is good approximation space of the real velocity space.

- [1] C. Grandmont, M. Hillairet, Existence of global strong solutions to a beam-fluid interaction system *Arch. Ration. Mech. Anal*, **220**(3):1283–1333, 2016.
- [2] J. A San Martín, V. Starovoitov and M. Tucsnak, Global weak solutions for the two-dimensional motion of several rigid bodies in an incompressible viscous fluid *Arch. Ration. Mech. Anal.*, 161(2):113–147, 2002.

## Numerical solution of the viscous flows in a network of thin tubes: equations on the graph

 <u>Frédéric Chardard</u><sup>1</sup>, Éric Canon<sup>1</sup>, Grigory Panasenko<sup>1,2</sup>, Olga Štikonienė<sup>2</sup>
 <sup>1</sup> Univ Lyon, UJM-Saint-Etienne, CNRS, Institute Camille Jordan UMR 5208, SFR MODMAD FED 4169, F-42023, Saint-Etienne, France
 <sup>2</sup> Institute of Applied Mathematics, Vilnius University, Vilnius, Lithuania eric.canon@univ-st-etienne.fr, frederic.chardard@univ-st-etienne.fr, grigori.panasenko@univ-st-etienne.fr, olga.stikoniene@mif.vu.lt

A non-stationary flow in a network of thin tubes is considered. Its one-dimensional approximation was proposed in a paper by G.Panasenko and K.Pileckas [1]. It consists of a set of equations with weakly singular kernels, on a graph, for the macroscopic pressure. A new difference scheme for this problem is proposed. Several variants are discussed. Stability and convergence are studied theoretically and numerically. Numerical results are compared to the direct numerical solution of the full dimension Navier-Stokes equations.

More details about the kernels will be discussed in the talk by Éric Canon: On weakly singular kernels arising in equations set on a graph, modelling a flow in a networth of thin tubes.

#### References

[1] Panasenko G. P., Pileckas K., Flows in a tube structure: equation on the graph, *Journal of Mathematical Physics*, **55**: 081505, 2014.

## Personalized Computational Modeling of Transapical Mitral Valve Repair

<u>Gediminas Gaidulis</u><sup>1</sup>, Audrius Aidietis<sup>2</sup>, Rimantas Kačianauskas<sup>1</sup> <sup>1</sup> Vilnius Gediminas Technical University, Vilnius, Lithuania, gediminas.gaidulis@vgtu.lt, rimantas.kacianauskas@vgtu.lt

<sup>2</sup> Vilnius University Hospital Santaros Klinikos, Vilnius, Lithuania, audrius.aidietis@santa.lt

Development of the computational model for the simulation of transapical mitral valve (MV) repair is presented. Transapical MV repair with neochordae implantation is a novel surgical technique, allowing beating-heart correction of mitral regurgitation caused by chordae tendineae rupture through a minimally-invasive approach [1].

In the present study, personalized computational modeling strategy to perform virtual transapical MV repair was used to evaluate the effects of such procedure on the post-repair MV function. Structural finite element model of the MV decoupled from the blood flow and composed of shell and truss elements with nonlinear hyperelastic material properties was created based on pre-operative echocardiographic image data of a patient with ruptured chordae tendineae who underwent transapical MV repair procedure. Patient-specific kinematic boundary conditions were incorporated, and time-dependent physiologic transvalvular pressure was applied to approximate the decoupled behavior of blood.

Virtual transapical MV repair was performed by positioning neochordae following the reallife surgery process executed by surgeons. A dynamic finite element analysis in a time frame between end-diastole and peak systole was conducted to evaluate the function of the MV before and after virtual repair. Computational MV simulation and modeling results provided quantitative information about the neochordae contribution to the MV function improvement and presented the potential of such modeling strategy to have practical value in scenarios of clinical relevance.

#### References

 Lancellotti P., Radermecker M., Durieux R., Modine T., Oury C., Fattouch K., Transapical beating-heart chordae implantation in mitral regurgitation: a new horizon for repairing mitral valve prolapse, *Journal of Thoracic Disease*, 8(12):E1665–E1671, 2016.

## On the Motion of a Rigid Body with a Liquid-Filled Interior Cavity

<u>Giovanni P. Galdi</u> University of Pittsburgh, U.S.A., galdi@pitt.edu

Problems involving the motion of a rigid body with a cavity filled with a viscous liquid are of fundamental interest in several applied areas of research, including dynamics of flight, space technology, and geophysical problems. Besides its important role in physical and engineering disciplines, the motion of these coupled systems generates a number of mathematical questions, which are intriguing and challenging at the same time. They are principally due to the occurrence of different and coexisting dynamic properties, such as the dissipative nature of the liquid, and the conservative character of some components of the angular momentum of the coupled system as a whole. One important characteristic of this interaction is that the presence of the liquid can dramatically influence the motion of the rigid body and produce a "stabilizing" effect that, in some cases, can even bring the coupled system to rest. Objective of this talk is to present a rather complete mathematical analysis of the dynamics of such systems and provide a rigorous explanation of some of the most relevant observed phenomena.

## Mixed-Dimensional Coupling for Time-Dependent Wave Problems

Dan Givoli<sup>1</sup>, Hanan Amar<sup>2</sup>, Daniel Rabinovich<sup>3</sup> Technion, Haifa, Israel, <sup>1</sup>givolid@technion.ac.il <sup>2</sup>hananamar2007@gmail.com <sup>3</sup>aedaniel@technion.ac.il

In recent years, along with the increase in the size and complexity of computational models, the scenario of mixed-dimensional (LowD-HighD) coupling has drawn a lot of attention. Fields of application where this scenario is of special interest include, among others, (a) blood-flow analysis, (b) hydrological and geophysical flow models, and (c) elastic structures. We apply the proposed coupling methods to wave problems governed by the scalar wave equation and by the equations of elastodynamics. Panasenko *et al.* have developed an asymptotic-variational approach for such problems, under static conditions. See, e.g., [1].

After splitting the given problem into a 2D part and a 1D part, thus creating a hybrid 2D-1D model, we need to consider the interface conditions imposed at the continuous level. We impose pointwise continuity on the wave function u (which also implies that u is uniform along the interface), and continuity in the average sense on the "flux." We prove that the hybrid model involving these two interface conditions is well-posed.

We shall consider three coupling methods: (1) the Panasenko coupling method, where the *u*-continuity condition is imposed strongly and the flux-continuity condition is imposed weakly [2]; (2) the Nitsche coupling method, where both the *u*-continuity condition and the flux-continuity condition are imposed weakly [3]; and (3) the Dirichlet-to-Neumann (DtN) coupling method, where information is passed at each time-step between the 2D model and the 1D model by using the numerically-obtained DtN mapping of the 1D model on the interface.

By experimenting with various numerical examples, we shall investigate the performance of the three methods, estimate the errors that each of them generates, and compare between them in accuracy and efficiency.

- [1] G. P. Panasenko, The Partial Homogenization: Continuous and Semi-discretized Versions, *Math. Models & Meth. Appl. Sci.* 8: 1183–1209, 2007.
- [2] H. Amar and D. Givoli, Mixed-Dimensional Modeling of Time-Dependent Wave Problems Using the Panasenko Construction, *J. Theor. Comput. Acoust.* **26**(03): 1850034-1–29, 2018.
- [3] H. Amar and D. Givoli, Mixed-Dimensional Coupling for Time-Dependent Wave Problems Using the Nitsche Method, *Comp. Meth. Appl. Mech. & Engng.* **349**: 213–250, 2019.

## Asymptotic analysis of the viscous flow through a pipe and the derivation of the Darcy-Weisbach law

Eduard Marušić-Paloka University of Zagreb, Zagreb, Croatia, emarusic@math.hr

Darcy-Weisbach formula is used to compute the pressure drop of the fluid in the pipe, due to the friction against the wall. Because of its simplicity, the Darcy-Weisbach formula become widely accepted by engineers and is used for laminar as well as the turbulent flows through pipes, once the method to compute the mysterious friction coefficient was derived. Particularly in the second half of the 20-th century. Formula is empiric and our goal is to derive it from the basic conservation law, via rigorous asymptotic analysis. We consider the case of the laminar flow but with significant Reynolds number. In case of the perfectly smooth pipe, the situation is trivial, as the Navier-Stokes system can be solved explicitly via the Poiseuille formula leading to the friction coefficient in the form 64/Re. For the rough pipe some effects of the roughness appear in the friction coefficient. We start from the Navier-Stokes system in the pipe with periodically corrugated wall and derive an asymptotic expansion for the pressure and for the velocity. We use the homogenization techniques and the boundary layer analysis. The derived approximation is then justified by rigorous error estimate in the norm of the appropriate Sobolev space, using the energy formulation and classical a priori estimates. Our method leads to the formula for the friction coefficient. The formula involves numerical resolution of the appropriate boundary layer problems. However, theoretical analysis characterising their nature can be done without solving them.

#### References

[1] E.Marušić-Paloka, Effective fluid behavior and the Darcy-Weisbach law, accepted for publication in SIAM J.Appl.Math.

## The FSI framework for the numerical modeling of the arterial blood flow and its application in the clinical case scenario

Edgaras Misiulis, Algis Džiugys, Robertas Navakas, Gediminas Skarbalius Lithuanian Energy Institute, Kaunas, Lithuania, edgaras.misiulis@lei.lt, algis.dziugys@lei.lt, robertas.navakas@lei.lt, gediminas.skarbalius@lei.lt

We continue on the previously proposed fluid-structure interaction (FSI) framework for the numerical modeling of the arterial blood and introduce a new method for the fiber structure generation, which generates the uniformly directed fibers in the artery wall [1]. We show its applicability in the clinical case scenario. The aim of our selected clinical case is to provide the basis for the accuracy improvement of the innovative, non-invasive intracranial pressure measurement (nICP) method [2]. In order to reach our aim, we used the arterial blood flow FSI framework together with the COMSOL Multiphysics<sup>(R)</sup> to model the blood flow in the patient-specific ophthalmic artery (OA), which is the main sensor, used in the nICP method. It is a strongly coupled FSI problem due to the similarity between the density of the blood and the density of the artery wall. We validated the implementation of the prestress, the Holzapfel-Gasser-Ogden (H-G-O) constitutive artery wall model and of our developed fiber structure generation method. Finally, we determined the dependencies of the characteristic parameters - the blood flow velocity and the lumen crosssectional area of the OA. The standard deviation measure was used to evaluate the oscillations in the time domain of the difference of characteristic parameters between the specific cross-sectional areas in the intracranial and extracranial segments of the OA. The obtained dependencies of the characteristic parameters provide the basis for the improvement of the nICP method's accuracy.

- [1] Misiulis E., Džiugys A., Navakas R., Petkus V. A comparative study of methods used to generate the arterial fiber structure in a clinically relevant numerical analysis, *International Journal for Numerical Methods in Biomedical Engineering*, (*in press*).
- [2] Siaudvytyte L., Januleviciene I., Ragauskas A., Bartusis L., Siesky B., Harris A. Update in intracranial pressure evaluation methods and translaminar pressure gradient role in glaucoma, *Acta Ophthalmologica*, 93(1):9–15, 2015.

## **Existence and regularity for weak solutions for a fluid** interacting with a non-linear shell in 3D

Boris Muha<sup>1</sup>, Sebastian Schwarzacher<sup>2</sup> <sup>1</sup> Department of Mathematics Faculty of Science, University of Zagreb, Croatia, borism@math.hr <sup>2</sup> Department of Mathematical Analysis, Charles University, Prague, Czech Republic, schwarz@karlin.mff.cuni.cz

We study the unsteady incompressible Navier Stokes equations in three dimensions interacting with a non-linear flexible shell of Koiter Type. The latter one constitutes a moving part of the boundary of the physical domain of the fluid. This leads to a coupled system of non-linear PDEs where the moving part of the boundary is an unknown of the problem. We study weak solutions to the corresponding fluid-structure interaction (FSI) problem. The known existence theory for weak solutions is extended to non-linear Koiter shell models. This is achieved by introducing new methods that allow us to prove higher regularity estimates for the shell by transferring damping effects from the fluid dissipation. The regularity result depends on the geometric constitution alone and is independent of the approximation procedure; hence it holds for arbitrary weak solutions.

## **Biomechanics of arterial thrombosis**

<u>Dmitry Nechipurenko</u><sup>1</sup>, Valeria Kaneva<sup>2</sup>, Mikhail Panteleev<sup>2</sup>, Fazly Ataullakhanov<sup>3</sup>
 <sup>1</sup> Lomonosov Moscow State University, Faculty of Physics, Moscow, Russia, ne4ipur@gmail.com
 <sup>2</sup> Center for Theoretical Problems of Physicochemical Pharmacology, Russian Academy of Sciences, Moscow, Russia
 <sup>3</sup> Dmitry Rogachev National Medical Research Center of Pediatric Hematology, Oncology and Immunology, Moscow, Russia

Formation of arterial thrombus in response to vessel wall injury may significantly decrease the local blood flow and thus lead to life-threatening conditions, such as myocardial infarction and stroke. The reason why some injuries result in formation of stable occlusive thrombus, compromising the blood flow, is poorly understood. It has been shown that arterial thrombus is heterogeneous: dense thrombus core, rich in tightly packed and activated platelets, is covered with highly dynamic shell, wherein platelets are loosely packed and poorly activated [1]. During normal response to the injury, thrombus shell is removed by the flow, leaving stable thrombus core, which covers the injury site. Understanding the mechanisms responsible for observed dynamics of arterial thrombus is of major importance for the field. The consolidation of thrombus core, called thrombus contraction, represents an active process, dependent on actomyosin complexes inside platelets. Recently we demonstrated that thrombus contraction results in mechanically-driven redistribution of weakly-adhesive dying platelets to thrombus exterior [2]. To investigate whether thrombus contraction and mechanical redistribution of dying platelets may impact the dynamics the arterial thrombus, we first developed particle-based in silico model of thrombus formation, which reproduces dynamics of thrombus shell observed in vivo [3]. Simulation results predicted the range of critical forces mediating platelet-platelet interactions in the shell, which was further used for computational analysis of thrombus stability under various conditions.

This work was supported by the Russian Foundation for Basic Research grant 19-51-15004.

- [1] Stalker T.J., Traxler E.A. et al, Hierarchical organization in the hemostatic response and its relationship to the platelet-signaling network, *Blood*, **121**:1875–1885, 2013.
- [2] Nechipurenko D.Y., Receveur N. et al, Clot contraction drives the translocation of procoagulant platelets to thrombus surface, *ATVB*, **39**:37–47, 2019.
- [3] Trifanov P.V., Kaneva V.N. et al, Developing Quasi-Steady Model for Studying Hemostatic Response Using Supercomputer Technologies, *Supercomputing Frontiers and Innovations*, 5:57–72, 2018.

## Effects of rough boundary and boundary conditions on the lubrication process with micropolar fluid

Igor Pažanin Department of Mathematics, Faculty of Science, University of Zagreb, Bijenička 30, Zagreb, Croatia, pazanin@math.hr

The lubrication theory is mostly concerned with the behavior of a lubricant flowing through a narrow gap. Motivated by the experimental findings from the tribology literature, we take the lubricant to be a micropolar fluid and study its behavior in a thin domain with specific rough boundary. We first consider (commonly used) simple zero boundary condition for the microrotation [1], and, after that, we impose (physically relevant) dynamic boundary conditions [2]. Using rigorous asymptotic analysis of the corresponding 3D boundary value problems, we propose simplified mathematical models acknowledging the roughness-induced effects and the effects of the boundary conditions on the macroscopic flow. In case of dynamic boundary conditions, we study numerically the influence of the roughness on the performance of a linear slider bearing. The numerical results clearly indicate that the use of the rough surfaces may contribute to enhance the mechanical performance of such device.

This is a joint work with M. Bonnivard (Université Paris Diderot) and F.J. Suárez-Grau (Universidad de Sevilla).

- [1] Pažanin I., Suárez-Grau F.J., Analysis of the thin film flow in a rough domain filled with micropolar fluid, *Computers & Mathematics with Applications*, **68**:1915–1932, 2014.
- [2] Bonnivard M., Pažanin I., Suárez-Grau F.J., Effects of rough boundary and nonzero boundary conditions on the lubrication process with micropolar fluid, *European Journal of Mechanics -B/Fluids*, 72:501–518, 2018.

## Mathematical models for the heart and the circulation

Alfio Quarteroni Politecnico di Milano, Milan, Italy EPFL, Lausanne, Switzerland alfio.quarteroni@epfl.ch

Mathematical models based on first principles can describe the interaction between electrical, mechanical and fluid-dynamical processes occurring in the heart, as well as the coupling with the external circulation. This is a classical multi-physics problem featuring multi-scale solutions in space and time. Appropriate numerical strategies need to be devised to allow for an accurate and computationally effective simulation of these processes in both physiological and pathological regimes. This presentation will address some of these issues and a few representative applications of clinical interest.

Acknowledgment: The work presented in this talk is part of the project iHEART that has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 740132).

## Rigorous justification of the asymptotic model describing a nonsteady micropolar fluid flow through a thin curved pipe

Marko Radulović, Igor Pažanin

Department of Mathematics, Faculty of Science, University of Zagreb, Zagreb, Croatia, mradul@math.hr, pazanin@math.hr

In this talk, we present the rigorous derivation of the asymptotic model describing a nonsteady flow of a micropolar fluid through a thin (or long) curved pipe [1].

Using Germano's reference system to describe the pipe's geometry, we write the micropolar fluid equations in curvilinear coordinates, construct a second order asymptotic expansion, study the boundary layers in space and provide the rigorous justification of the model via error estimate between the original solution and the constructed asymptotic approximation.

This work is an extension of the model derived for the nonsteady flow of a micropolar fluid through a thin undeformed pipe [2]. Due to the practical importance of the micropolar fluid model in biomedicine and blood flow modeling, the talk will be focused on mathematical as well as engineering aspects of the model.

- [1] Pažanin I., Radulović M., Asymptotic analysis of the nonsteady micropolar fluid flow through a curved pipe, *Applicable Analysis*, doi:10.1080/00036811.2018.1553036, 1–48, 2018.
- [2] Beneš M., Pažanin I., Radulović, M., Rigorous derivation of the asymptotic model describing a nonsteady micropolar fluid flow through a thin pipe, *Computers and Mathematics with Applications*, **76**(9): 2035–2060, 2018.

## Cardiovascular modeling and simulations. Applications to some clinical studies

Adélia Sequeira Department of Mathematics and CEMAT, Instituto Superior Técnico, University of Lisbon, Lisbon, Portugal, adelia.sequeira@tecnico.ulisboa.pt

Cardiovascular diseases, such as heart attack and strokes, are the major causes of death in developed countries, with a significant impact in the cost and overall status of healthcare. Understanding the fundamental mechanisms of the pathophysiology and treatment of these diseases are matters of the greatest importance around the world. This gives a key impulse to the progress in mathematical and numerical modeling of the associated phenomena governed by complex physical laws, using adequate and fully reliable *in silico* settings.

In this talk we describe some mathematical models of the cardiovascular system and comment on their significance to yield realistic and accurate numerical results, using stable, reliable and efficient computational methods. Results on the simulation of some image-based patient-specific clinical cases will also be presented.

## Spatial asymptotics of time dependent Stokes problems

Maria Specovius<sup>1</sup>, Konstantinas Pileckas<sup>2</sup> <sup>1</sup>University of Kassel, Kassel, Germany, specovi@mathematik.uni-kassel.de <sup>2</sup>Vilnius University, Vilnius, Lithuania, konstantinas.pileckas@mif.vu.lt

While there are numerous papers about time decay of solutions to parabolic problems there is considerably less known about the spatial decay of solutions to these problems in unbounded domains. Here we present some ideas how to obtain such results for time dependent Stokes problems in a layer-like domain. The main problem here is not to find more or less explicit expressions for the asymptotic terms which can be done by judicious guessing but to derive precise estimates in appropriate weighted Sobolev spaces, in order to justify the asymptotic decomposition.

#### References

 Pileckas K., Specovius-Neugebauer M., Spatial behavior of solutions to the time periodic Stokes system in a three dimensional layer, *Journal of Differential Equations*, 263: 6317– 6346, 2017.

## Control of fluid-structure interaction problems with medical applications

<u>Ruxandra Stavre</u> Institute of Mathematics "Simion Stoilow", Romanian Academy, Bucharest, Romania Ruxandra.Stavre@imar.ro

We present two control problems for fluid-elastic structure interaction inspired directly from real life.

The first control problem is related to the blood flow in venous insufficiency. When a vein loses its elasticity, blood stagnation and reflux may occur; these phenomena lead to medical complications such as leg edema and varicose ulcers. A treatment for venous insufficiency is represented by compression stockings that compensate the lack of elasticity of the vein walls. We associate to this physical phenomenon a mathematical model that proposes a method for determining an exterior compression which realizes a blood flow without or with minimum reflux.

The second control problem concerns the analysis of the blood pressure variations with respect to the changes of the blood viscosity. There exist several medical causes which produce abnormal blood pressure, such as diseases that determine the formation of atheromatous plaques in arteries. We introduce and study a mathematical model which provides a blood viscosity distribution that leads to a desired configuration of the blood pressure.

# Incompressible viscous fluid interacting with a nonlinear thermoelastic plate

<u>Srđan Trifunović</u><sup>1</sup>, Ya-Guang Wang<sup>2</sup> School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai, China, <sup>1</sup>sergej1922@gmail.com <sup>2</sup>ygwang@sjtu.edu.cn

Here we deal with a nonlinear interaction problem between an incompressible viscous fluid and a thermoelastic plate. The fluid and the plate are fully coupled and the plate only moves in the lateral direction. The fluid is governed by the three-dimensional incompressible Navier-Stokes equations, while the plate is governed by a system of two (scalar) equations - the nonlinear plate equation, describing the balance of forces, and the heat equation, describing the distribution of the heat in the plate. The nonlinearity in the plate equation corresponds to nonlinear elastic force in various plate models, such as:

- 1. Semilinear case: Kirchhoff, von Kármán and Berger plates (see [1]);
- 2. Quasilinear case: The nonlinearity takes the form of  $\Delta((\Delta w)^3)$ , where w is the lateral displacement of the plate (see [2]);

We prove the existence of a weak solution by constructing a novel hybrid approximation scheme that is inspired by the time discretization+operator spliting approximation scheme developed by Muha and Čanić (see [3]). We split the problem into two sub-problems, one piece-wise stationary for the fluid and one continuous in time and spanned in a finite Galerkin basis for the structure. In order to ensure the convergence of approximate solutions, we introduce a certain dependence between the number of basis functions for the structure sub-problem and the time discretization step.

In the quasilinear case, to pass the convergence in the nonlinear term, we implement the compensated compactness method which relies on the maximal monotonicity property of the nonlinear function. This allows us to pass to the limit without having to obtain additional spatial regularity.

- I. Chueshov, S. Kolbasin, Long-time dynamics in plate models with strong nonlinear damping, *Comm. Pure App. Anal.*, 11(2012), 659–674.
- [2] I. Lasiecka, S. Maad, A. Sasane, Existence and Exponential Decay of Solutions to a Quasilinear Thermoelastic Plate System, *Nonlin. Diff. Eq. and App. NoDEA*, 15(2008), 689–715.
- [3] B. Muha, S. Čanić, Existence of a weak solution to a nonlinear fluid-structure interaction problem modeling the flow of an incompressible, viscous fluid in a cylinder with deformable walls, *Arch. Ration. Mech. Anal.*, 207(2013), 919–968.

## Method of asymptotic partial decomposition with discontinuous junctions. Finite volume implementation

Marie-Claude Viallon<sup>1</sup>, Grigory Panasenko<sup>1,2</sup>

<sup>1</sup> University of Lyon, Institute Camille Jordan, Saint-Etienne, France, canon@univ-st-etienne.fr, grigory.panasenko@univ-st-etienne.fr

<sup>2</sup> Institute of Applied Mathematics, Vilnius University Vilnius, Lithuania

Method of asymptotic partial decomposition of a domain (MAPDD) proposed and justified earlier in [1] for thin domains (rod structures, tube structures consisting of a set of thin cylinders) generates some special interface conditions between two-dimensional or three-dimensional and one-dimensional parts. In the case of the heat equation these conditions insure the continuity of the solution and continuity in average of the normal flux. In the case of fluid mechanics these conditions prescribe a pre-computed Poiseuille-type shape of a solution at the interface. Although these conditions give a high order precision for the non-steady heat equation and for three dimensional Navier-Stokes equations in tube structures it may be undesirable for the reason of numerical implementation to impose the Dirichlet type conditions at the cross sections of junctions. In the present paper we introduce and justify the alternative junction conditions allowing discontinuity of the solution but preserving continuity of the average over cross section. The finite volume method is applied for the numerical solution of the hybrid dimension model. Theoretical results are proved and numerical experiments are presented.

- [1] Panasenko G., *Multi-Scale Modelling for Structures and Composites*, Springer, Dordrecht, 2005.
- [2] Panasenko G. Viallon M.-C., Finite volume implementation of the method of asymptotic partial domain decomposition for the heat equation on a thin structure. *Russian Journal of Math. Phys.*, 22 (2):237–263, 2015.

### On the Equations of Nonlinear Single-Phase Poroelasticity

C. J. van Duijn<sup>1</sup>, Andro Mikelić<sup>2</sup>

<sup>1</sup>Darcy Center, Eindhoven University of Technology, Department of Mechanical Engineering and Utrecht University, Department of Earth Sciences, The Netherlands, C.J.v.Duijn@TUE.nl
<sup>2</sup> Univ Lyon, Université Claude Bernard Lyon 1, CNRS UMR 5208, Institut Camille Jordan, 43 blvd. du 11 novembre 1918, F-69622 Villeurbanne cedex, France, mikelic@univ-lyon1.fr

In this talk we present a study of the equations of nonlinear poroelasticity derived from mixture theory. They describe the quasi-static mechanical behaviour of a fluid saturated porous medium. The nonlinearity arises from the compressibility of the fluid and from the dependence of porosity and permeability on the divergence of the displacement. We point out analytical difficulties with the model. In our approach we discretize the quasi-static formulation in time and first consider the corresponding incremental problem. For this, we prove existence of a solution using Brézis' theory of pseudo-monotone operators. Generalizing Biot's free energy to the nonlinear setting we construct a Lyapunov functional for the model and prove stability. It allows constructing bounds that are uniform with respect to the time step. In the case when dissipative interface effects between the fluid and the solid are taken into account, we consider the continuous time case in the limit when the time step tends to zero. This yields existence of a weak free energy solution.

- van Duijn C.J., Mikelić A., Wick T., A monolithic phase-field model of a fluid-driven fracture in a nonlinear poroelastic medium, *Mathematics and Mechanics of Solids*, 24(5): 1530–1555, 2019.
- van Duijn C.J., Mikelić A., Mathematical Theory of Nonlinear Single-Phase Poroelasticity, Preprint hal-02144933, Lyon June 2019, https://hal-univ-lyon1.archives-ouvertes.fr/hal-02144933

### A Multi-scale Model for Magnetorheological Suspensions

Bogdan Vernescu<sup>1</sup>, Grigor Nika<sup>2</sup>

<sup>1</sup>Worcester Polytechnic Institute, Worcester, MA, USA, vernescu@wpi.edu <sup>2</sup>Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany, grigor.nika@wias-berlin.de

Using the homogenization method we obtain a model describing the behavior of a suspension of solid magnetizable particles in a viscous non-conducting fluid. The model generalizes the Neuringer and Rosensweig [1], [2] model for quasistatic phenomena, extended in [3]. We derive the macroscopic constitutive properties explicitly in terms of the solutions of local problems. Unlike in the case of suspensions [4] without the presence of electric or magnetic fluid, the effective fluid is non-Newtonian, and the effective coefficients have a nonlinear dependence on the volume fraction, due to the formation of chain structures. The velocity profiles computed for some simple flows, exhibit an apparent yield stress and the flow profiles resemble those for a Bingham-type fluid, with similarities to the results for electrorheological fluids [5].

- [1] Nueringer J. L., Rosensweig R. E., Ferrohydrodynamics, *Physics of Fluids*, 7:1927–1937, (1964).
- [2] Rosensweig, R. E., Ferrohydrodynamics, Dover Publications, 2014.
- [3] Shliomis, M. I., Effective viscosity of magnetic suspensions, *Sov J Exp Theor Physics*, 34: 1291–1294, (1972).
- [4] Lipton R., Vernescu B., Homogenization of two-phase emulsions, *Proc Roy Soc Edinburgh*, 124A: 1119–1134, (1994).
- [5] Vernescu B., Multiscale analysis of electrorheological fluids, *Inter J Modern Physics B*, **16**: 2643–2648 (2002).
- [6] Mei C. C., Vernescu B., Homogenization Methods for Multiscale Mechanics, World Scientific, 2010.

## Fluid flow in a flexible pipe with layered elastic walls

German Zavorokhin<sup>1</sup>, Sergei Nazarov<sup>2</sup>, Vladimir Kozlov<sup>3</sup> <sup>1</sup> St. Petersburg Department of V. A. Steklov Mathematical Institute, St.Petersburg, Russia, zavorokhin@pdmi.ras.ru <sup>2</sup> Institute of Mechanical Engineering Problems RAS, St.Petersburg, Russia, srgnazarov@yahoo.co.uk <sup>3</sup> Mathematics and Applied Mathematics, MAI, Linköping University, Linköping SE-58183, Sweden, vladimir.kozlov@liu.se

A two-dimensional model describing the elastic behaviour of the wall of a curved flexible pipe has been presented in [1]. The wall has a laminate structure consisting of several anisotropic layers of varying thickness and is assumed to be much smaller in thickness than the radius of the channel which itself is allowed to vary. This two-dimensional model takes the interaction of the wall with any surrounding or supporting material and the fluid flow into account and is obtained via a dimension reduction procedure. We study spectral problem associated with the time-periodic Stokes flow in the straight flexible pipe with elastic walls. In comparison with the Stokes problem for the pipe with rigid walls we prove that the Womersley flow analogue [2] does not exist for the pipe with elastic walls. We construct the leading term of the asymptotic expansion of the solution for the Stokes problem in the pipe with elastic walls [3].

- [1] Ghosh G., Kozlov V. A., Nazarov S. A., Rule D., A two-dimensional model of the thin laminar wall of a curvilinear flexible pipe, *The Quarterly Journal of Mechanics and Applied Mathematics*, **71**(3):349–367, 2018.
- [2] Womersley J. R., Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known, *J. Physiol.*, **127**(3):553–563, 1955.
- [3] Kozlov V. A., Nazarov S. A., Zavorokhin G. L., Fluid flow in a flexible pipe with layered elastic walls, *in preparation*, 2019.

International Conference *Multiscale Modeling in Fluid Mechanics and Fluid-Structure Interaction* Vilnius, Lithuania, October 7–11, 2019

## **Participants**

Aidietis Audrius audrius.aidietis@santa.lt

```
Ball John jb101@hw.ac.uk
Bertoglio Cristóbal c.a.bertoglio@rug.nl
```

```
Canon Éric eric.canon@univ-st-etienne.fr
Chardard Frédéric eric.canon@univ-st-etienne.fr
```

```
Gaidulis Gediminas gediminas.gaidulis@vgtu.lt
Galdi Giovanni P. galdi@pitt.edu
Givoli Dan givolid@technion.ac.il
Grandmont Celine celine.grandmont@inria.fr
```

Kaulakytė Kristina kaulakyte.kristina@gmail.com

```
Marušić-Paloka Eduard emarusic@math.hr
Mikelić Andro mikelic@univ-lyon1.fr
Misiulis Edgaras edgaras.misiulis@lei.lt
Muha Boris borism@math.hr
```

Nechipurenko Dmitry ne4ipur@gmail.com

```
Panasenko Grigory grigori.panasenko@univ-st-etienne.fr
Pažanin Igor pazanin@math.hr
Pileckas Konstantinas konstantinas.pileckas@mif.vu.lt
```

Quarteroni Alfio alfio.quarteroni@epfl.ch

Radulović Marko mradul@math.hr

```
Sequeira Adéliaadelia.sequeira@tecnico.ulisboa.ptSpecovius Mariaspecovi@mathematik.uni-kassel.deStavre RuxandraRuxandra.Stavre@imar.roŠtikonienė Olgaolga.stikoniene@mif.vu.lt
```

Trifunović Srđan sergej1922@gmail.com

Vernescu Bogdan vernescu@wpi.edu Viallon Marie-Claude canon@univ-st-etienne.fr

Zavorokhin German zavorokhin @pdmi.ras.ru

### **Index of Authors**

Aidietiene, Sigita, 1 Aidietis, Audrius, 1, 8 Amar, Hanan, 10 Ataullakhanov, Fazly, 14

Ball, John, 2 Bertoglio, Cristóbal, 3

Canon, Éric, 4, 5, 7 Casanova, Jean-Jérôme, 6 Chardard, Frédéric, 4, 5, 7

Drasutiene, Agne, 1 Džiugys, Algis, 12

Gaidulis, Gediminas, 8 Galdi, Giovanni P., 9 Givoli, Dan, 10 Grandmont, Céline, 6

Hillairet, Matthieu, 6

Kačianauskas, Rimantas, 8 Kaneva, Valeria, 14 Kozlov, Vladimir, 25

Marušić-Paloka, Eduard, 11 Mikelić, Andro, 23 Misiulis, Edgaras, 12 Muha, Boris, 13

Navakas, Robertas, 12

Nazarov, Sergei, 25 Nechipurenko, Dmitry, 14 Nika, Grigor, 24

Pažanin, Igor, 15, 17 Panasenko, Grigory, 4, 5, 7, 22 Panteleev, Mikhail, 14 Pileckas, Konstantinas, 5, 19

Quarteroni, Alfio, 16

Rabinovich, Daniel, 10 Radulović, Marko, 17 Rucinskas, Kestutis, 1

Schwarzacher, Sebastian, 13 Sequeira, Adélia, 18 Skarbalius, Gediminas, 12 Specovius, Maria, 19 Stavre, Ruxandra, 20 Štikonienė, Olga, 4, 5, 7

Trifunović, Srđan, 21

van Duijn, C. J., 23 Vernescu, Bogdan, 24 Viallon, Marie-Claude, 22

Wang, Ya-Guang, 21

Zakarkaite, Diana, 1 Zavorokhin, German, 25